

$$\frac{\rho C_p}{k} \frac{\partial \theta}{\partial t} = \frac{1}{a^2} \frac{\partial^2 \theta}{\partial \eta^2} + \frac{q'''}{k(T_0 - T_m)}$$

$$\frac{a^2}{\alpha} \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial \eta^2} + \frac{q''' a^2}{k(T_0 - T_m)}$$

$$\tau = \frac{\alpha t}{a^2} \Rightarrow t = \frac{\tau a^2}{\alpha}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} + G$$

$$\frac{\partial}{\partial t} = \frac{1}{(a^2/\alpha)} \frac{\partial}{\partial \tau}$$

$$-\frac{k(T_0 - T_m)}{a} \frac{\partial \theta}{\partial \eta} = h \cdot \theta(T_0 - T_m)$$

$$G = \frac{q''' a^2}{k(T_0 - T_m)}$$

$$-\frac{\partial \theta}{\partial \eta} = \frac{h a}{k} \cdot \theta$$

$$Bi = \frac{h a}{k} \quad \text{Biot}$$

$$-\frac{\partial \theta}{\partial \eta} = Bi \cdot \theta \quad \text{em } \eta = 1$$

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} + G$$

$$\frac{\partial \theta}{\partial \eta} = 0 \quad \text{em } \eta = 0$$

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$$\theta = 1 \quad \text{em } \tau = 0$$

$$\theta_{\text{médio}} \\ \theta_{\text{av}}(t) = \int_0^1 \theta(\eta, t) d\eta$$

$$\int_0^1 \frac{\partial \theta}{\partial \tau} d\eta = \int_0^1 \frac{\partial^2 \theta}{\partial \eta^2} d\eta + \int_0^1 G d\eta$$

$$\frac{d\theta_{\text{av}}(t)}{d\tau} = \frac{\partial \theta}{\partial \eta} \Big|_{\eta=1} - \frac{\partial \theta}{\partial \eta} \Big|_{\eta=0} + G$$

$$\frac{\partial}{\partial \tau} \int_0^1 \theta d\eta = \left[\frac{\partial \theta}{\partial \eta} \right]_0^1 + G$$

$$\frac{d\theta_{\text{av}}(t)}{d\tau} = -Bi \cdot \theta(1, \tau) + G$$

$\theta(1, \tau) = f(\theta_{\text{av}}(t))$ parâmetro concentrado clássico

$$\frac{d\theta_{\text{av}}(t)}{d\tau} = \left[\frac{\partial \theta}{\partial \eta} \right]_0^1 + G$$

$$\theta(1, \tau) \cong \theta_{\text{av}}(\tau) \quad Bi < 0,1$$

$$\frac{d\theta_{\text{av}}}{d\tau} = -Bi \cdot \theta_{\text{av}}(\tau) + G$$